2017-10-25

Week #5

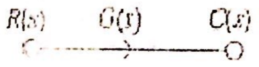
Reduction of Multiple Subsystem……………………………EE362

Multiple subsystems are represented in two ways:-

I- Block diagrams   II- Signal Flow Graphs (SFG).

Block diagrams are usually used for *frequency domain analysis* and *design* and *SFG for state space analysis.*

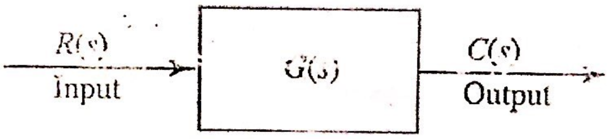
Block Diagram



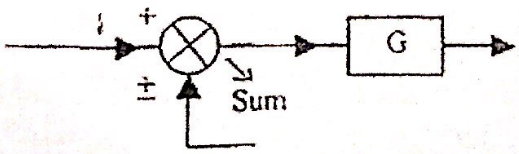
Signal Flow Graph

1- Block Diagrams

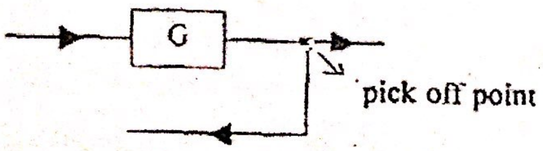
- The transfer function of any system can be represented by the **Block Diagram** as shown if figure below.



- Block diagrams consist of unidirectional, operational blocks that represent the transfer function of the variables of interest.
- The block diagram representation of a given system often can be reduced to a simplified block diagram with fewer blocks than original diagram.
- When multiple subsystems are interconnected, a few more schematic elements must be added to the block diagram. These new elements are **summing** junction and **pickoff** point. A signal may be taken from the output of a block and given to another block. This point from which the signal is tapped is known as **pickoff point**



Symbol of a Sum



Pick off point

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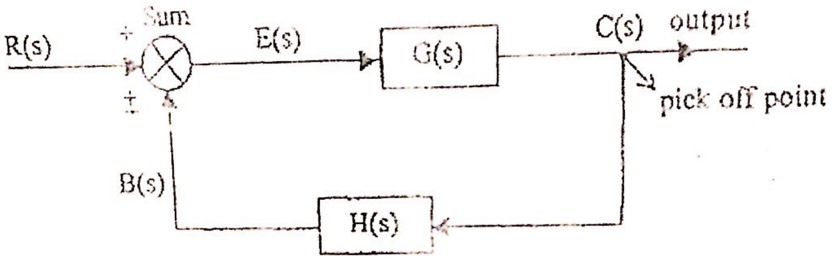
Closed loop system



$G(s) = \frac{C(s)}{E(s)}$; Forward path transfer function, or plant transfer function

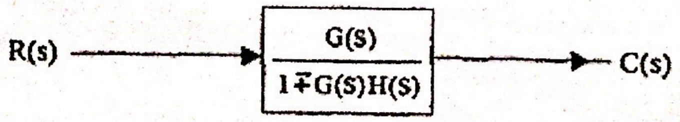
$H(s) = \frac{B(s)}{C(s)}$; Transfer function of the feedback elements

where, R(s); Reference input or desired output
C(s); Output or controlled variable
B(s); Feedback signal
E(s); Error signal
G(s) H(s); Loop transfer function

This closed loop system can be replaced by a single block by finding the transfer function $\frac{C(s)}{R(s)}$

$$C(s) = G(s)\,E(s)$$
$$E(s) = R(s) \pm B(s)$$
$$= R(s) \pm H(s)\,C(s)$$
$$C(s) = G(s)\,[R(s) \pm H(s)\,C(s)]$$
$$C(s)\,[1 \mp G(s)\,H(s)] = G(s)\,R(s)$$
$$\frac{C(s)}{R(s)} = \frac{G(s)}{1 \mp G(s)\,H(s)}$$

This transfer function can be represented by single block as shown in figure below

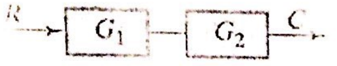
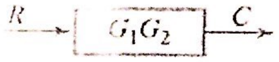
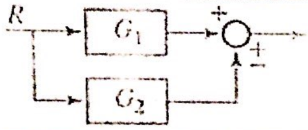
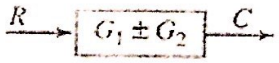
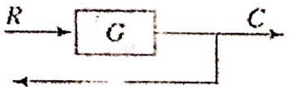
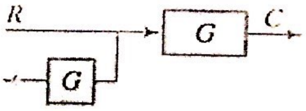
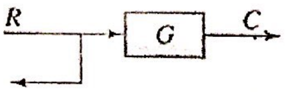
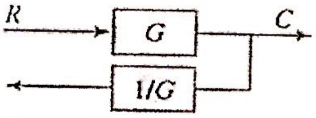
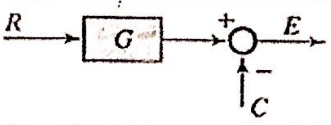
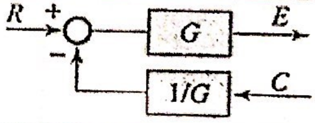
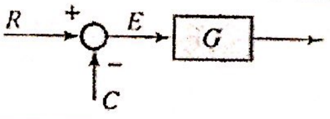
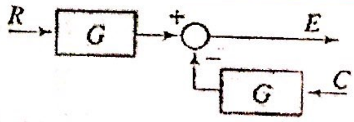
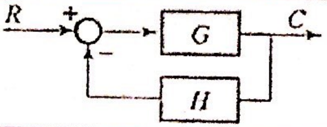
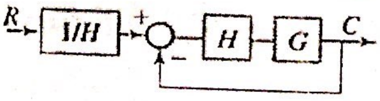
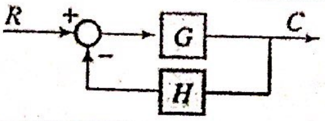
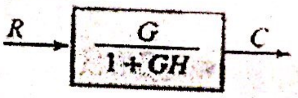


(**Note: if the feedback signal is added to the reference input the feedback is said to be positive feedback. On the other hand if the feedback signal is subtracted from the reference input, the feedback is said to be negative feedback).**

For the most common case of negative feedback.

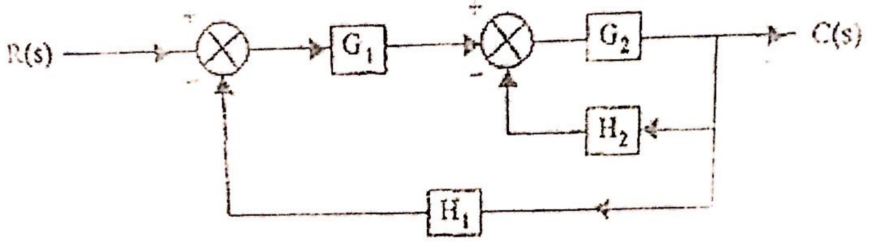
$$\frac{C(s)}{R(s)} \quad \frac{G(s)}{1+G(s)\,H(s)}$$

Diagram Reduction Manipulation

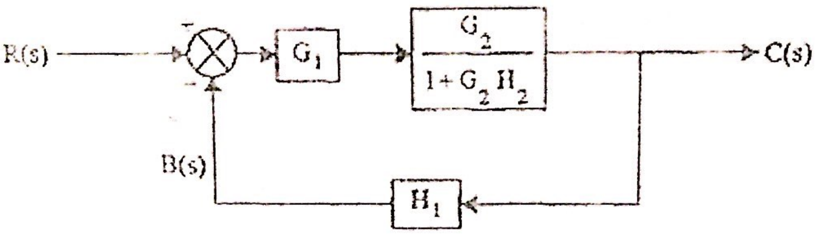
Original Block Diagram	Manipulation	Modified Block Diagram
	Cascaded elements	
	Addition or subtraction (eliminating auxiliary forward path)	
	Shifting of pickoff point ahead of block	
	Shifting of pickoff point behind block	
	Shifting summing point ahead of block	
	Shifting summing point behind block	
	Removing H from feedback path	
	Eliminating feedback path	

Ex

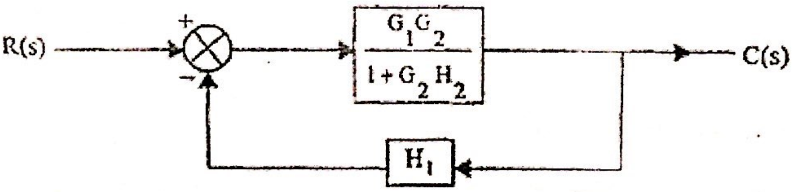
Find the overall transfer function of the system using block diagram reduction technique.



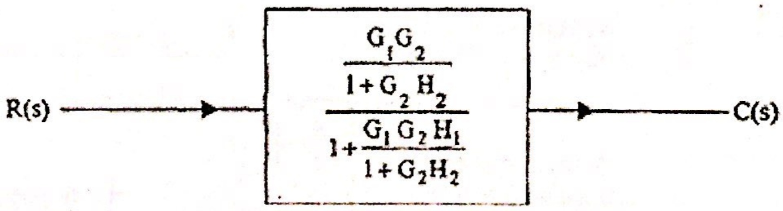
Step 1 : the feedback loop with G_2 and H_2 is replaced by a single block



Step 2 : The two blocks in the forward path are in cascade and can be replaced as shown



Step 3 : Finally, the loop is replaced by a single block

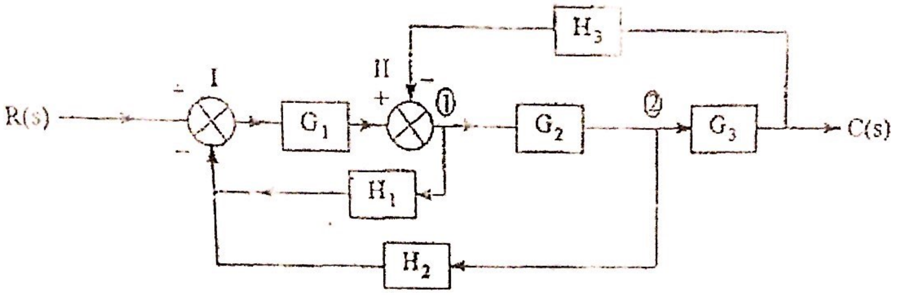


R(s)
$\frac{\frac{G_1G_2}{1+G_2H_2}}{1+\frac{G_1G_2H_1}{1+G_2H_2}}$
C(s)

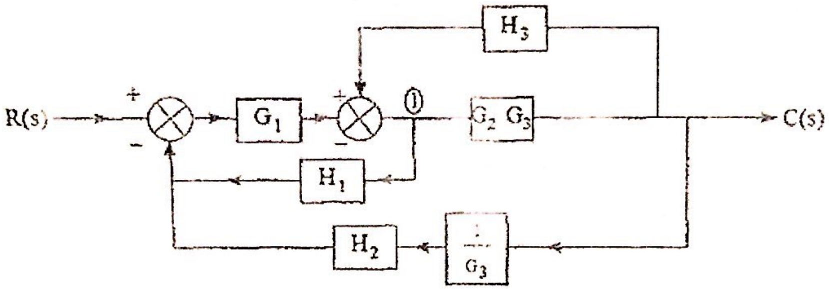
The transfer function can be simplified as,

$$\frac{C(s)}{R(s)}=\frac{G_1G_2}{1+G_2H_2+G_1G_2H_1}$$

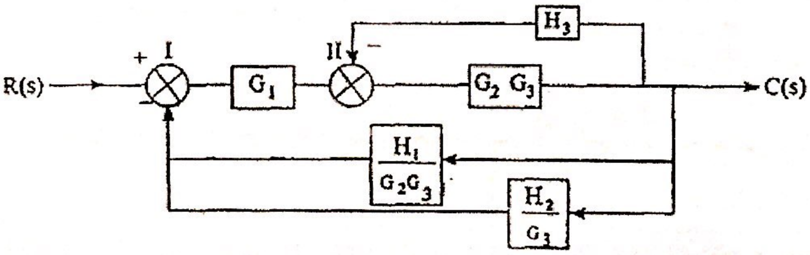
EX. Obtain the overall transfer function of the system shown in figure below.



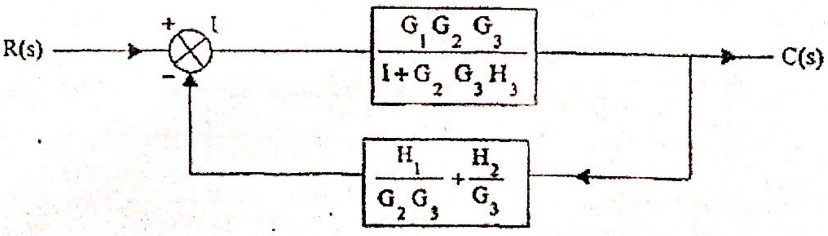
Step 1 : Moving the pick off point (2) to the right of block G_3 and combining blocks G_2 and G_3 in cascade, we have,



Step 2 : Moving the pick off point (1) to the right of block $G_2\ G_3$

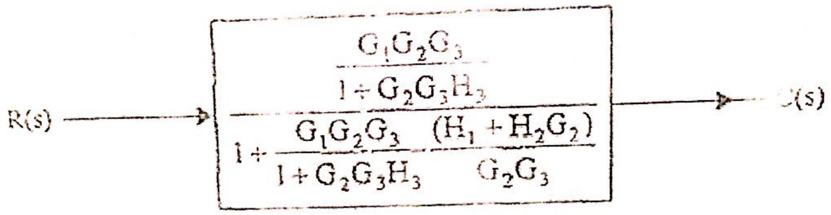


Step 3 : Absorbing the loop with $G_2\ G_3$ and H_3, and combining it with block G_1 in cascade, we have,



R(s)
+
I
-
$\frac{G_1G_2G_3}{1+G_2G_3H_3}$
$\frac{H_1}{G_2G_3}+\frac{H_2}{G_3}$
C(s)

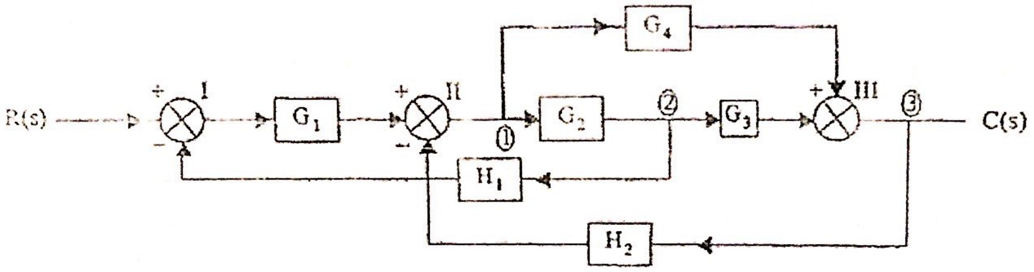
Step 4 : Finally the closed loop is absorbed and the simplified block is given by


R(s)
C(s)
$$\frac{\frac{G_1G_2G_3}{1+G_2G_3H_3}}{1+\frac{G_1G_2G_3}{1+G_2G_3H_3}\,\frac{(H_1+H_2G_2)}{G_2G_3}}$$

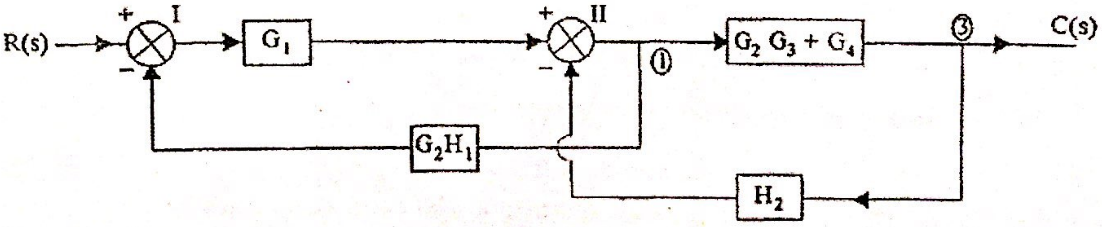
The transfer function of the system is,

$$\frac{C(s)}{R(s)}=\frac{G_1\,G_2\,G_3}{1+G_2\,G_3\,H_3+G_1\,H_1+G_1\,G_2\,H_2}$$

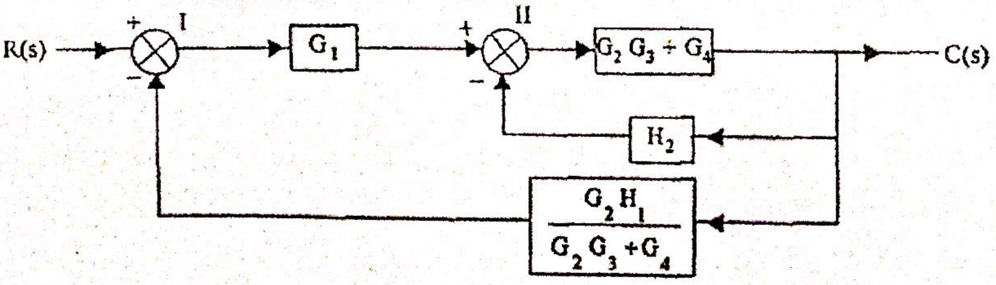
EX. Obtain the overall transfer function of the system shown in figure below



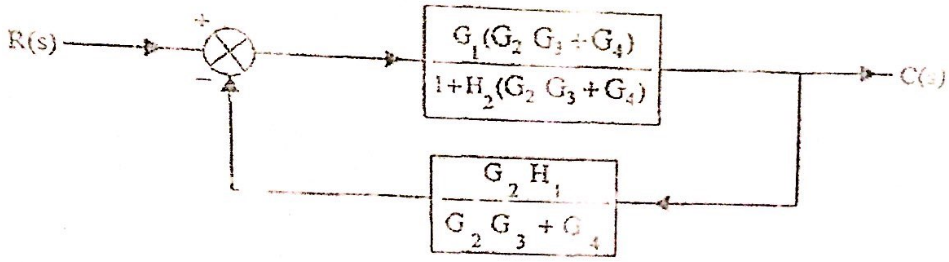
Step 1 : Move pick off point (2) to left of G_2 and combine G_2 G_3 in cascade. Further G_2 G_3 and G_4 have same inputs and the outputs are added at summer III. Hence they can be added and represented by a single block as shown



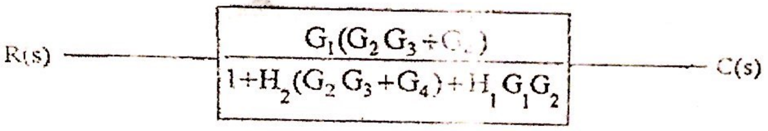
Step 2 : Moving the pick off point (1) to right of block (G_2 G_3 + G_4), we have



Step 3 : Absorbing loop with $(G_2 G_3 + G_4)$ and H_2


R(s)
+
−
$\frac{G_1(G_2 G_3 + G_4)}{1+H_2(G_2 G_3 + G_4)}$
C(s)
$\frac{G_2 H_1}{G_2 G_3 + G_4}$

The transfer function of the system is

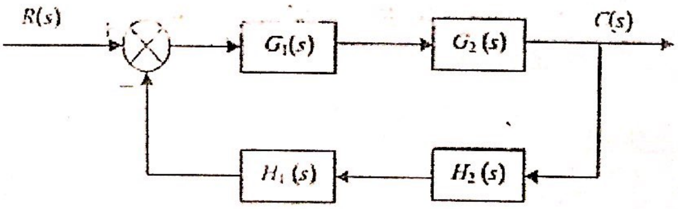

R(s)
$\frac{G_1(G_2 G_3 + G_4)}{1+H_2(G_2 G_3 + G_4) + H_1 G_1 G_2}$
C(s)

Key Commands (MATALB):

`Parallel` `series` `feedback` `append` `connect` `minreal`

Ex:

Reduce each of the block diagrams shown in the following figures to a single transfer function using MATLAB.



$$G1(s) = \frac{1}{s+1}, \quad G2(s) = \frac{s}{s+5},$$

$$H1(s) = \frac{1}{s}, \quad H2(s) = \frac{1}{s^2}$$

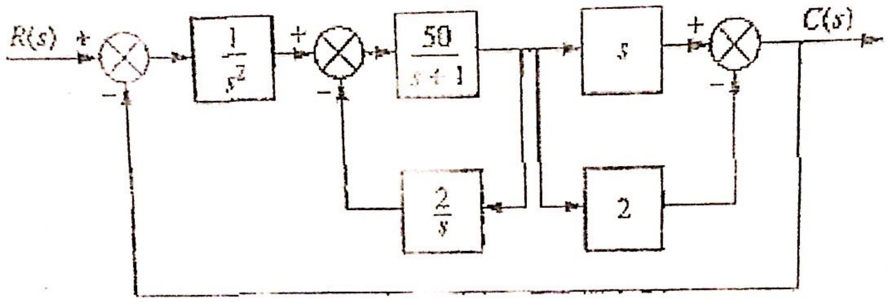
```
G1=tf([ 1],[1 1])        %G1=1/s+1
G2=tf([1 0],[1 5])       %G2=s/s+5
H1=tf(1,[1 0])           %H1=1/s
H2=tf([1],[1 0 0 ]       %H2=1/s^2
G= series(G1,G2)
H=series(H1,H2)
Ge= feedback(G,H)
minreal(Ge)
```

```
G1=tf([ 1],[1 1])        %G1=1/s+1
G2=tf([1 0],[1 5])       %G2=s/s+5
H1=tf(1,[1 0])           %H1=1/s
H2=tf([1],[1 0 0 ]       %H2=1/s^2
G= G1*G2
H= H1*H2
Ge= G/(1+G*H)
minreal(Ge)
```

```
Transfer function:
                   s^3
-------------------------------------
s^4 + 6 s^3 + 5 s^2 + 1.11e-016 s + 1
```

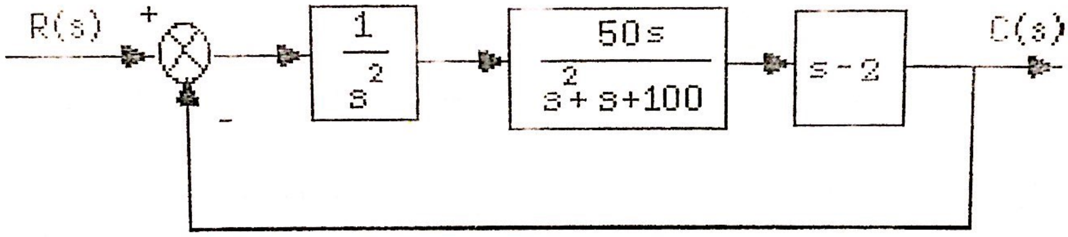
Ex Reduce the block diagram shown in figure below to a single transfer function $T(s) = C(s) / R(s)$. Use the following methods:-

a- Block diagram reduction. *b- MATLAB.*



a. Combine the inner feedback and the parallel pair.

b.



Multiply the blocks in the forward path and apply the feedback formula to get,

$$T(s) = \frac{50(s-2)}{s^3+s^2+150s-100}$$

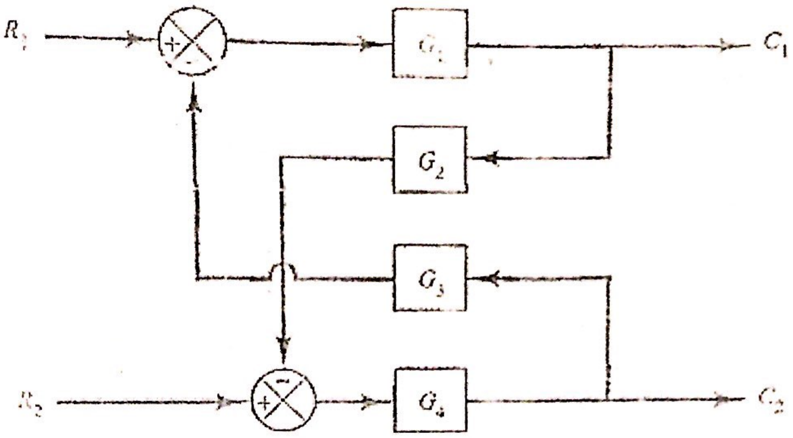
MATLAB code

```
clc
G1=tf(1,[1 0 0 ])           % G1 = 1/s2
G2=tf(50,[1 1])             % G2 = 50/s2+1
H2=tf(2,[1 0])              %  H2=2/s
G2=feedback(G2,H2)          % G2/(1+G2H2)
G3= tf([1 0],1)             % G3 = s
H3 =2;                      % H3 = 2
G3= parallel(G3,-H3)        %G3=G3-H3
G4= series(G1,G2);          % G1 * G2
G5= series(G3,G4);          % G3 * G4
T= feedback(G5,1)           % G5/(1+G5)
 minreal(T)
```

```
Transfer function:
     50 s - 100
-----------------------
s^3 + s^2 + 150 s - 100
```

EX

Derive $C_1(s)/R_1(s)$, $C_1(s)/R_2(s)$, $C_2(s)/R_1(s)$, and $C_2(s)/R_2(s)$. (In deriving outputs for $R_1(s)$, assume that $R_2(s)$ is zero, and vice versa.)



$$C_1 = G_1(R_1 - G_3C_2) \qquad (3\text{–}93)$$

$$C_2 = G_4(R_2 - G_2C_1) \qquad (3\text{–}94)$$

By substituting Equation (3–94) into Equation (3–93), we obtain

$$C_1 = G_1[R_1 - G_3G_4(R_2 - G_2C_1)] \qquad (3\text{–}95)$$

By substituting Equation (3–93) into Equation (3–94), we get

$$C_2 = G_4[R_2 - G_2G_1(R_1 - G_3C_2)] \qquad (3\text{–}96)$$

Solving Equation (3–95) for C_1, we obtain

$$C_1 = \frac{G_1R_1 - G_1G_3G_4R_2}{1 - G_1G_2G_3G_4} \qquad (3\text{–}97)$$

Solving Equation (3–96) for C_2 gives

$$C_2 = \frac{-G_1G_2G_4R_1 + G_4R_2}{1 - G_1G_2G_3G_4} \qquad (3\text{–}98)$$

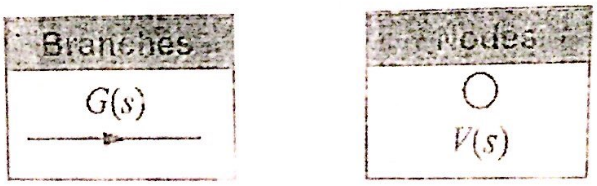
$$\begin{bmatrix} C_1 \\ C_2 \end{bmatrix} = \begin{bmatrix} \dfrac{G_1}{1 - G_1G_2G_3G_4} & -\dfrac{G_1G_3G_4}{1 - G_1G_2G_3G_4} \\ -\dfrac{G_1G_2G_4}{1 - G_1G_2G_3G_4} & \dfrac{G_4}{1 - G_1G_2G_3G_4} \end{bmatrix} \begin{bmatrix} R_1 \\ R_2 \end{bmatrix}$$

$$\frac{C_1(s)}{R_1(s)} = \frac{G_1}{1 - G_1G_2G_3G_4}, \qquad \frac{C_1(s)}{R_2(s)} = -\frac{G_1G_3G_4}{1 - G_1G_2G_3G_4}$$

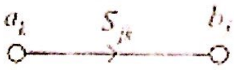
$$\frac{C_2(s)}{R_1(s)} = -\frac{G_1G_2G_4}{1 - G_1G_2G_3G_4}, \qquad \frac{C_2(s)}{R_2(s)} = \frac{G_4}{1 - G_1G_2G_3G_4}$$

2- Signal Flow Graph(SLG)

- Alternative to block diagram.
- Consists only branches (systems), and nodes (signals)



- A signal S_{jk} traveling along a branch between nodes a_k and b_j is multiplied by the gain of that branch

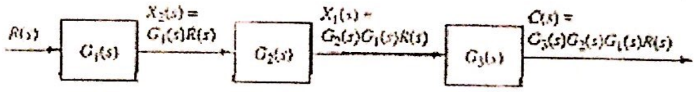


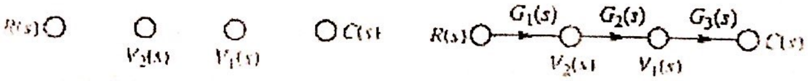
That is,

$$bj = S_{jk}\, a_k$$

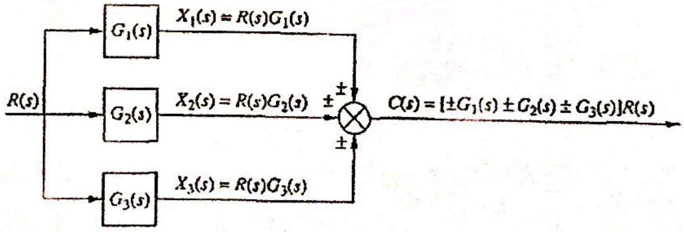
Converting Common Block Diagrams to Signal-Flow Graphs

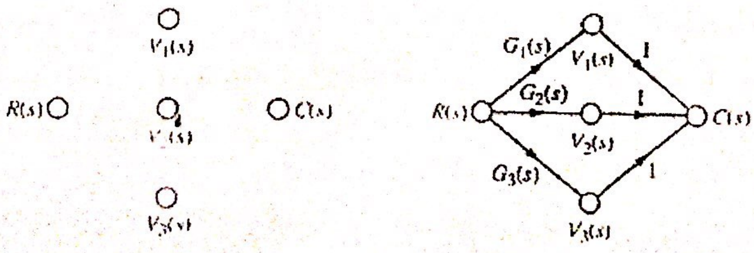
1- Series



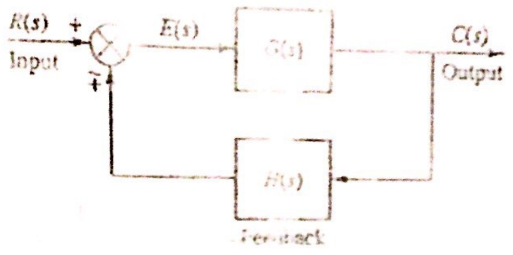


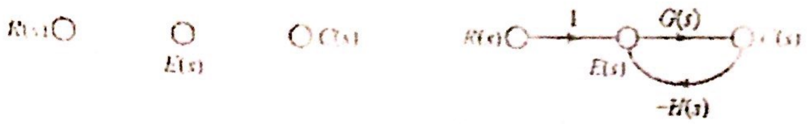
2- Parallel



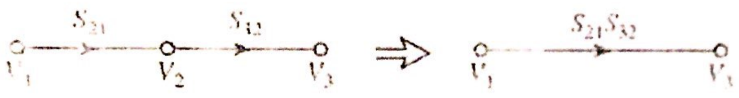


3- Feedback



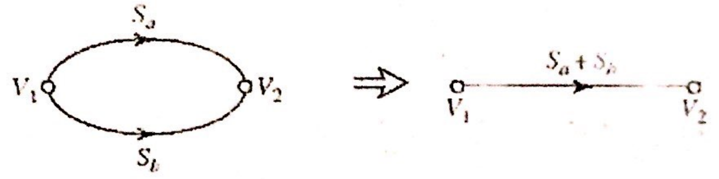


Decomposition Rules of Signal Flow Graphs



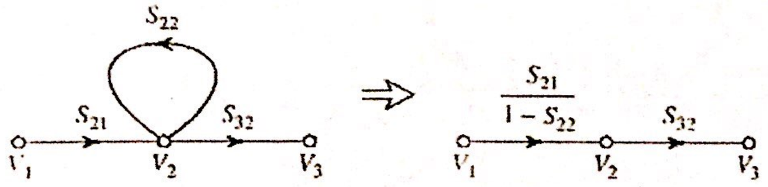
(a)

(1) series rule



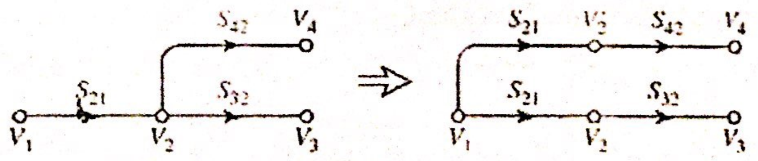
(b)

(2) parallel rule



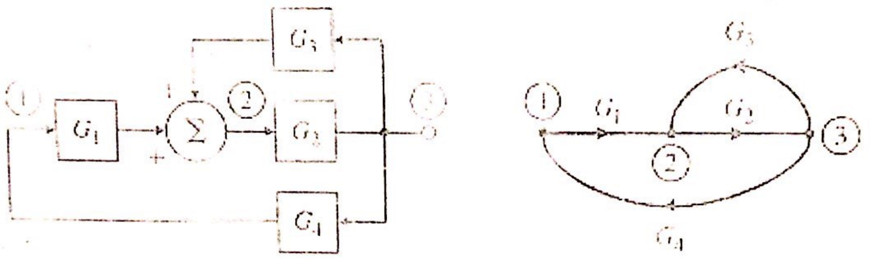
(c)

(3) self-loop rule

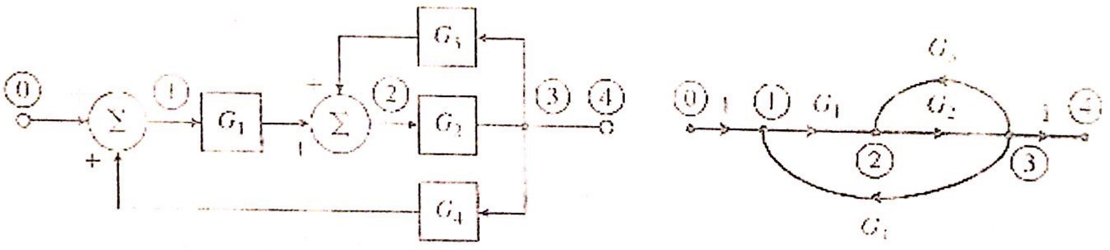


(4) splitting rule

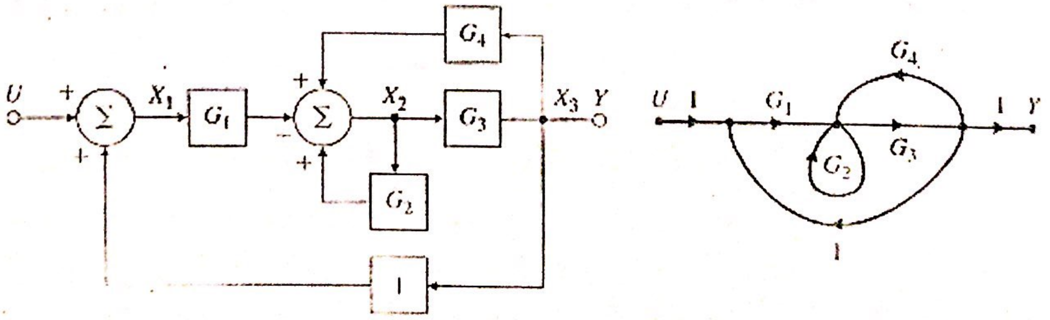
Ex Convert the block diagram of figure below to a signal-flow graph.



(a)

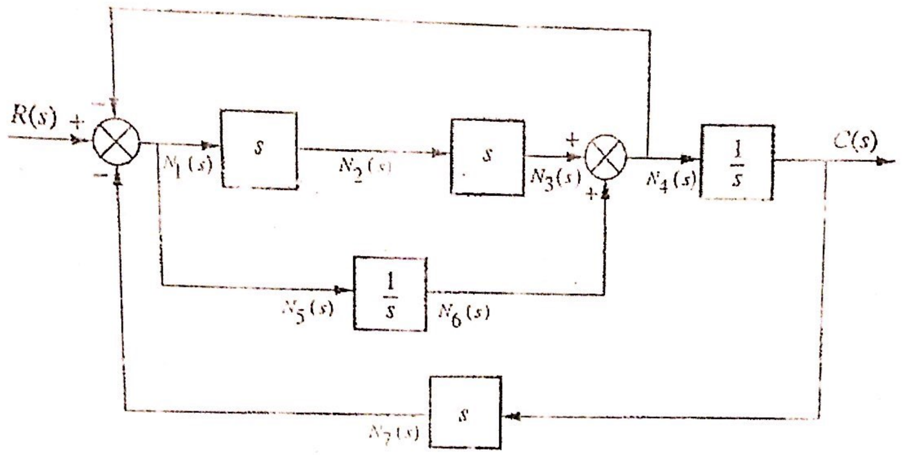


(b)

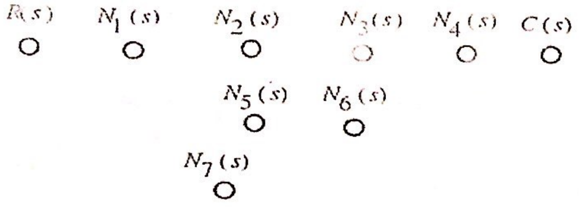


(c)

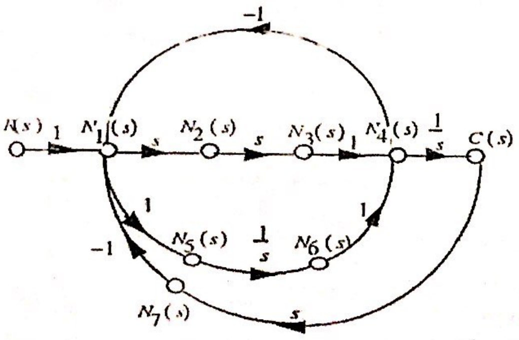
Ex: Convert the following block diagram to a signal flow graph.



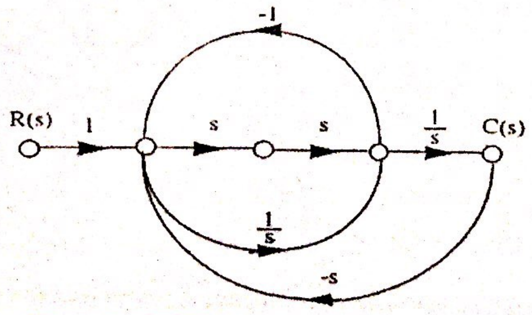
Draw nodes.



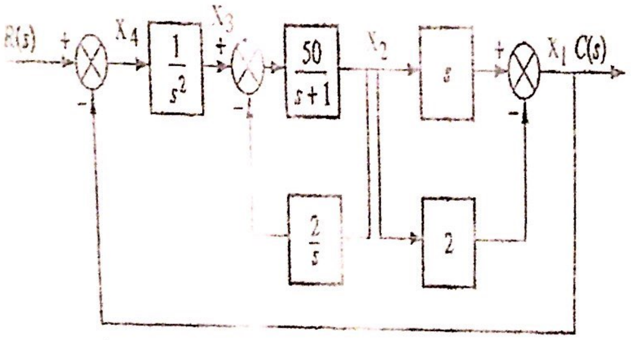
Connect nodes and label subsystems.

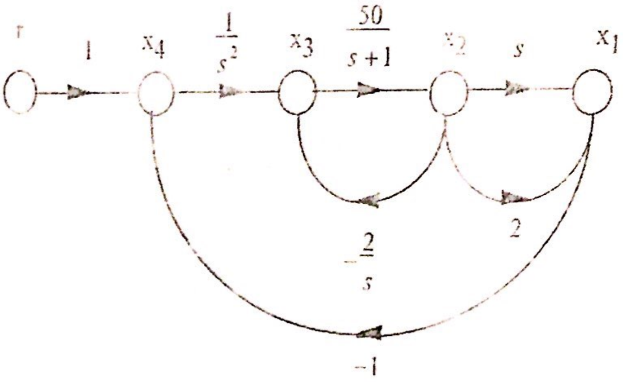


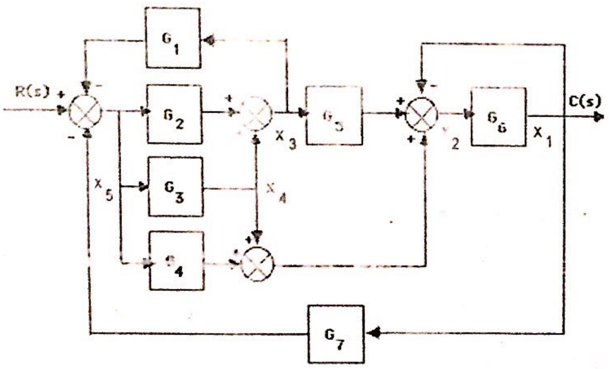
Eliminate unnecessary nodes.

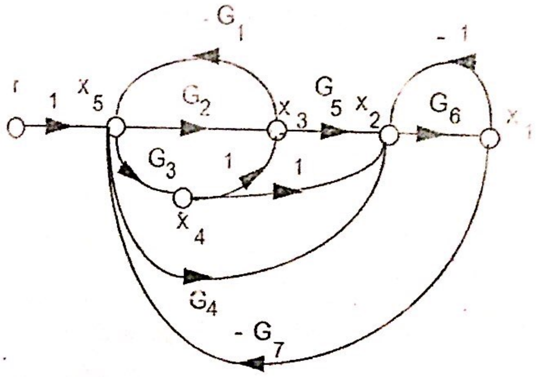


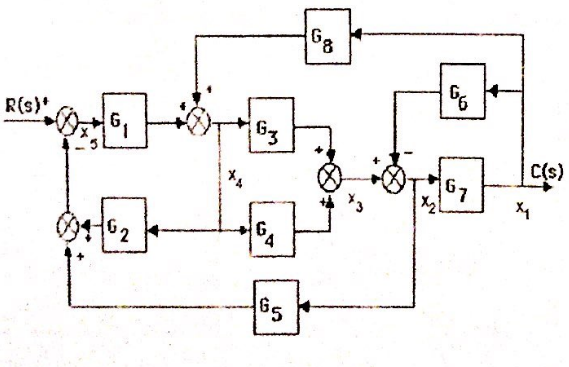
Ex Label signals and draw a signal- flow graph for the block diagrams shown in the following figures.

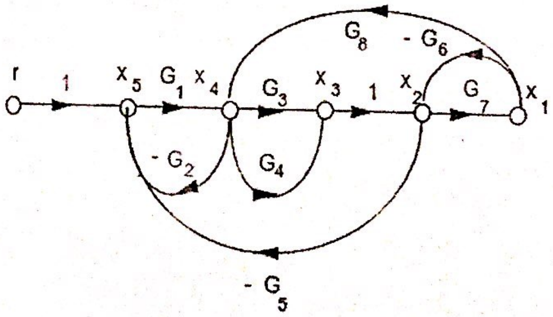




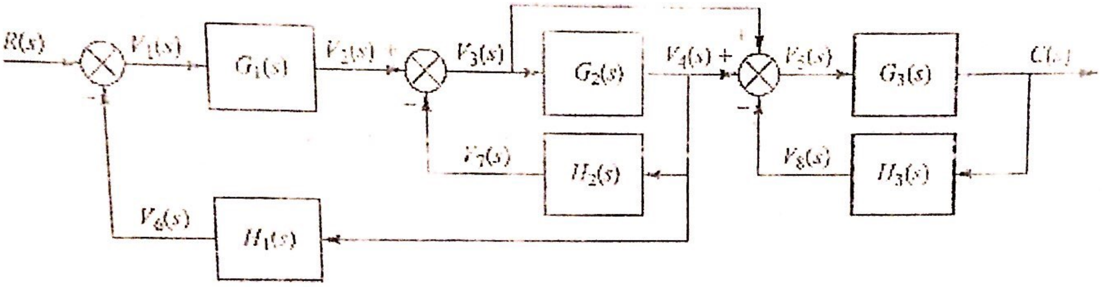




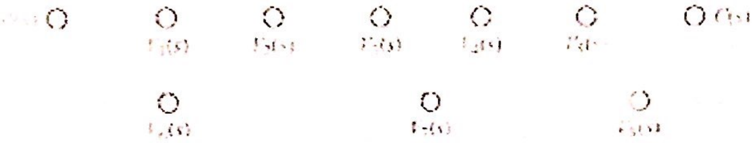




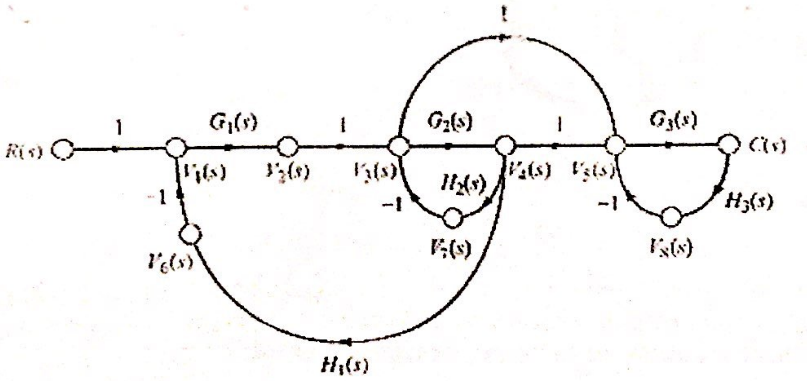
Ex Convert the block diagram of figure below to a signal-flow graph.



Draw the signal nodes



Interconnect the signal nodes with system branches



Mason's Rule

We discussed how to reduce block diagrams to single transfer functions. Now we are ready to discuss a technique for reducing signal-flow graphs to single transfer functions that relate the output of a system to its input.

The transfer function, C(s)/R(s), of a system represented by a signal-flow graph is

$$\boxed{G(s) = \frac{C(s)}{R(s)} = \frac{\sum_{k} T_k \Delta_k}{\Delta}}$$

Δ: determinate of the graph.

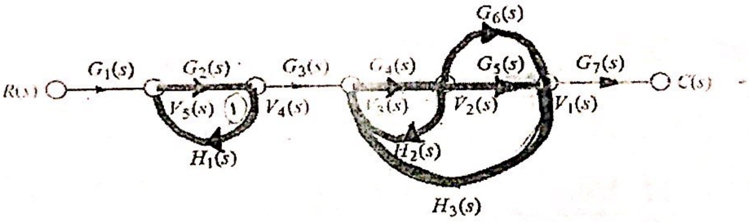
Δ_k: Cofactor of the Path Tk

Removing the loops that touch Path K from Δ.

- ☐ *k* = number of forward path
- ☐ *Tk* = the *k*th forward path gain
- ☐ Δ = 1 – (Σ loop gains) + (Σ non-touching loop gains taken two at a time) – (Σ non-touching loop gains taken three at a time).
- ☐ Δ*k* = 1 – (loop-gain which does not touch the forward path)

Mason's Rule – definitions

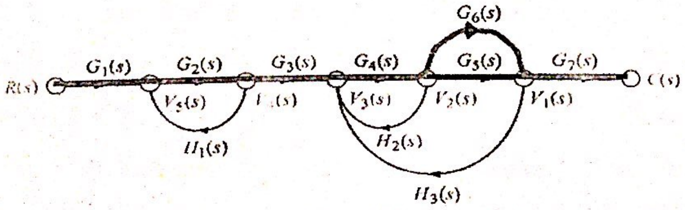
① ▪ **Loop gain** – the product of branch gains found by traversing a path that starts at a node and ends at the same node



L1. $G_2(s)H_1(s)$ L2. $G_4(s)H_2(s)$

L3. $G_4(s)G_5(s)H_3(s)$ L4. $G_4(s)G_6(s)H_3(s)$

② ▪ **Forward path** – the product of gains found by traversing a path from the input node to the output node of the signal-flow graph

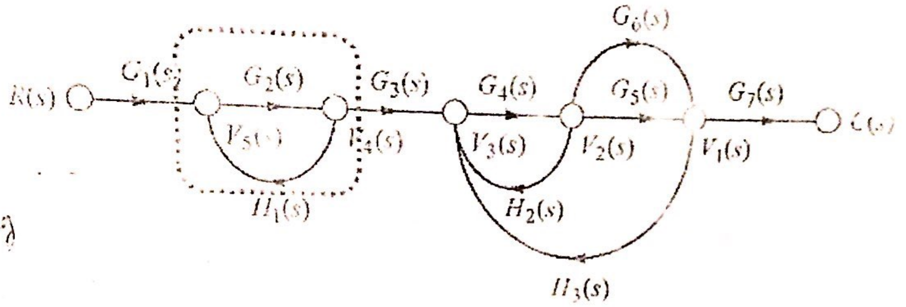


F1 – $G_1(s)G_2(s)G_3(s)G_4(s)G_5(s)G_7(s)$ ✓

F2 – $G_1(s)G_2(s)G_3(s)G_4(s)G_6(s)G_7(s)$ ✓

③ ▪ **Non-touching loop** – Loops that do not have any nodes in common

▪ **Non-touching loop gain** – The product of loop gains from non-touching loops taken two, three, four, or more at a time.



Non-touching ↖

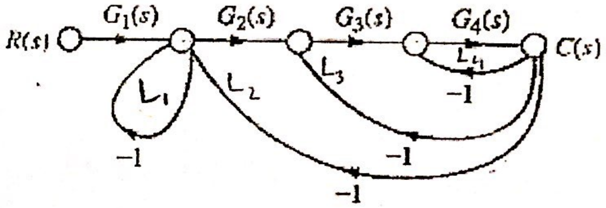
NT1. $[G_2(s)H_1(s)][G_4(s)H_2(s)]$

NT2. $[G_2(s)H_1(s)][G_4(s)G_5(s)H_3(s)]$

NT3. $[G_2(s)H_1(s)][G_4(s)G_6(s)H_3(s)]$

EX.

Using Mason's rule, find the transfer function, T(s) = C(s)/R(s), for the system represented in figure below.



$P1 = G_1G_2G_3G_4$
$L1 = -1$
$L2 = -G_2G_3G_4$
$L3 = -G_3G_4$
$L4 = -G_4$

$\Delta = 1-(L_1+L_2+L_3+L_4) +(L_1L_3 + L_1L_4) = 1 – (-1 -G_2G_3G_4 -G_3G_4 -G_4)+(G_3G_4+G_4)$
$\Delta = 2+ G_2G_3G_4 +2G_3G_4 +2G_4$

$\Delta 1 = 1 - 0$

$$T(s) = \frac{C(s)}{R(s)} = \frac{P_1\Delta_1}{\Delta} = \frac{G1G2G3G4}{2 + G2G3G4 + 2G3G4 + 2G4}$$

EX

Using Mason's rule find T.F

- The Paths connecting the input R(s) and output Y(s) are

$P_1 = G_1 G_2 G_3 G_4$

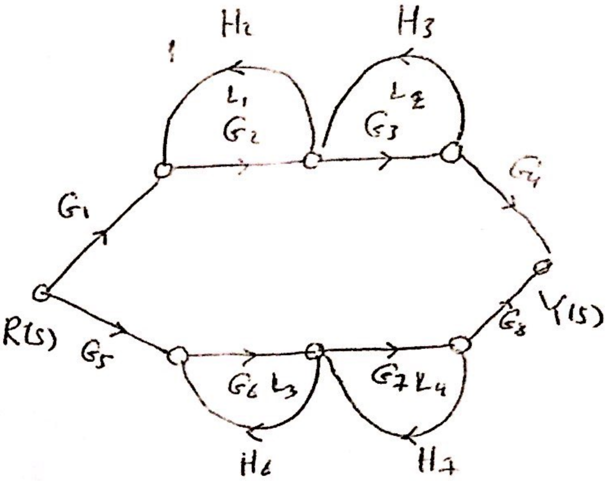
$P_2 = G_5 G_6 G_7 G_8$

$L_1 = G_2 H_2$

$L_2 = G_3 H_3$

$L_3 = G_6 H_6$

$L_4 = G_7 H_7$



$$T(s) = \frac{P_1\Delta_1 + P_2\Delta_2}{\Delta}$$

$$\Delta = 1 - (L_1 + L_2 + L_3 + L_4) + (L_1L_3 + L_1L_4 + L_2L_3 + L_{24})$$

$\Delta_1 \longrightarrow$ Removing the loops that touch Path 1 from Δ

$\Longrightarrow (L_1 = 0, L_1 = 0)$

$$\Delta_1 = 1 - (L_3 + L_4)$$

$\Delta_2 \longrightarrow$ Removing the loops that touch Path 2 from Δ

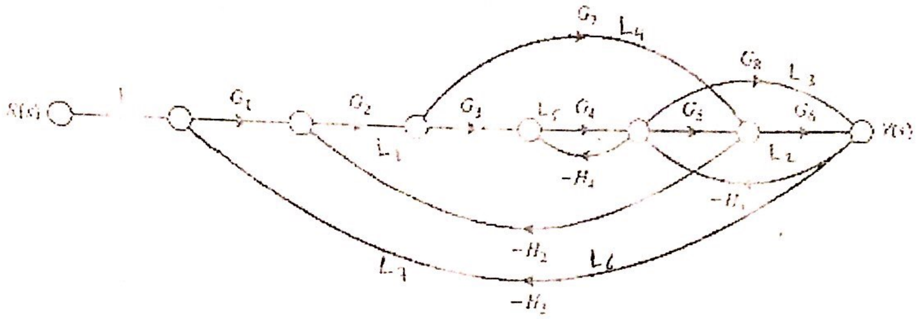
$(L_3 = L_4 = 0)$

$$\Delta_2 = 1 - (L_1 + L_2)$$

$$\therefore T(s) = \frac{P_1\Delta_1 + P_2\Delta_2}{\Delta}$$

EX.

Using Mason's rule, find the transfer function, T(s) = C(s)/R(s) for the system represented in figure below.



The forward paths are

$P_1 = G_1G_2G_3G_4G_5G_6$ $\qquad$ $P_2 = G_1G_2G_7G_6$ $\qquad$ $P_3 = G_1G_2G_3G_4G_8$

The feedback loops are

$L_1 = -G_2G_3G_4G_5H_2$ $\qquad$ $L_2 = -G_5G_6H_1$ $\qquad$ $L3 = -G_8H_1$ $\qquad$ $L_4 = -G_7H_2G_2$
$L_5 = -G_4H_4$ $\qquad$ $L_6 = -G_1G_2G_3G_4G_5G_6H_3$ $\qquad$ $L_7 = -G_1G_2G_7G_6H_3$
$L_8 = -G_1G_2G_3G_4G_8H_3$

Loop L_5 does not touch loop L_4 or Loop7, and loop L_3 does not touch loop L_4, but all other loops touch. Therefore the determinate is

$$\Delta = 1 - (L_1 + L_2 + L_3 + L_4 + L_5 + L_6 + L_7 + L_8) + (L_5L_7 + L_5L_4 + L_3L_4)$$

The cofactors are .

$\Delta_1 = \Delta_3 = 1$ and $\Delta_2 = 1 - L_5 = 1 + G_4H_4$

Finally, the transfer function is

$$T(s) = \frac{C(s)}{R(s)} = \frac{P_1\,\Delta_1 + P_2\,\Delta_2 + P_3\,\Delta_3}{\Delta}$$

Signal-Flow Graphs of State Equations

EX. Draw a signal-flow graph for the following state and output equations:

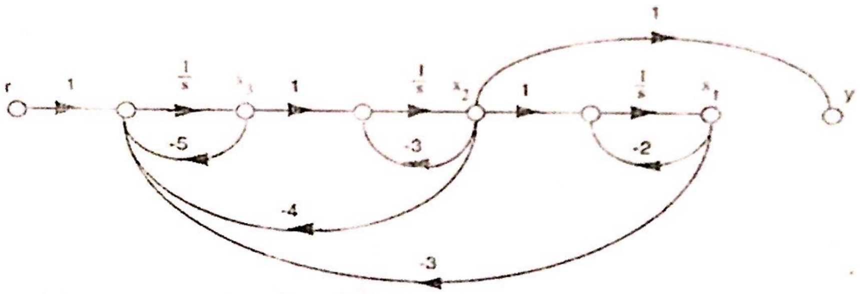
$$\dot{x}=\begin{bmatrix}-2 & 1 & 0\\ 0 & -3 & 1\\ -3 & -4 & -5\end{bmatrix}x+\begin{bmatrix}0\\0\\1\end{bmatrix}r \quad , \quad y=\begin{bmatrix}0 & 1 & 0\end{bmatrix}x$$

The state equations are,

$$\dot{x}_1=-2x_1+x_2 \qquad \dot{x}_2=-3x_2+x_3 \qquad \dot{x}_3=-3x_1-4x_2-5x_3+r$$

$$y=x_2$$

Drawing the signal-flow diagram from the state equations yields



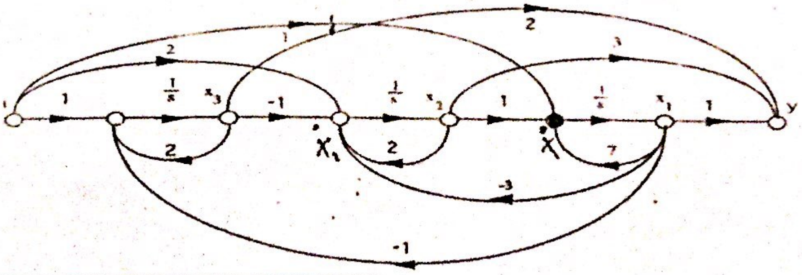
EX. Draw a signal-flow graph for the following state and output equations:

$$\dot{\mathbf{x}}=\begin{bmatrix}7 & 1 & 0\\ -3 & 2 & -1\\ -1 & 0 & 2\end{bmatrix}\mathbf{x}+\begin{bmatrix}1\\2\\1\end{bmatrix}r$$

$$y=\begin{bmatrix}1 & 3 & 2\end{bmatrix}\mathbf{x}$$

$$\dot{x}_1=7x_1+x_2+r \qquad \dot{x}_2=-3x_1+2x_2-x_3+2r$$

$$\dot{x}_3=-x_1+2x_3+r \qquad y=x_1+3x_2+2x_3$$



1- Phase Variable Form

$$\frac{C(s)}{R(s)} = G(s) = \frac{10}{s^3 + 24s^2 + 191s + 504}$$

$$\dddot{c} + 24\ddot{c} + 191\dot{c} + 504c = 10r$$

Let, $c = x_1$

$\dot{c} = x_2$

$\ddot{c} = x_3$

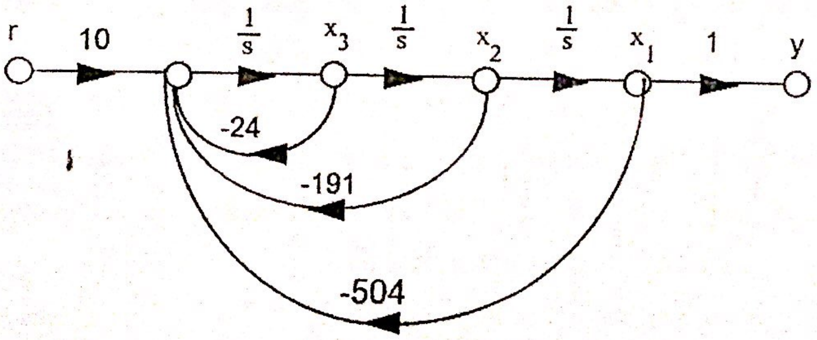
Therefore,

$\dot{x}_1 = x_2$

$\dot{x}_2 = x_3$

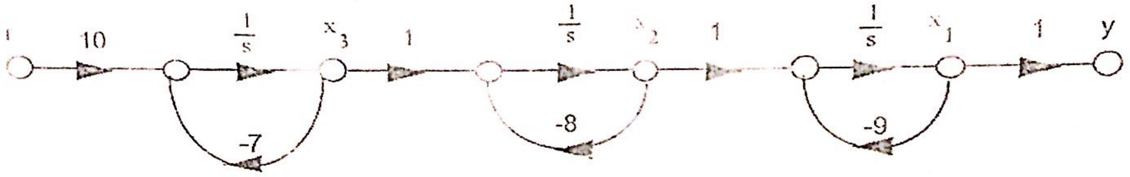
$\dot{x}_3 = -504x_1 - 191x_2 - 24x_3 + 10r$

$$\dot{x}\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -504 & -191 & -24 \end{bmatrix} x + \begin{bmatrix} 0 \\ 0 \\ 10 \end{bmatrix} r \quad , \quad y = [1 \quad 0 \quad 0]x$$



2- Cascaded Form

$$\frac{C(s)}{R(s)} = \frac{10}{s^3+24s^2+191s+504} = \left(\frac{10}{s+7}\right)\left(\frac{1}{s+8}\right)\left(\frac{1}{s+9}\right)$$



Therefore,

$$\dot{x}_1 = -9x_1 + x_2$$

$$\dot{x}_2 = -8x_2 + x_3$$

$$\dot{x}_3 = -7x_3 + 10r$$

$$y = x_1$$

$$\dot{x} = \begin{bmatrix} -9 & 1 & 0 \\ 0 & -8 & 1 \\ -0 & 0 & -7 \end{bmatrix} x + \begin{bmatrix} 0 \\ 0 \\ 10 \end{bmatrix} r \quad , \quad y = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix} x$$

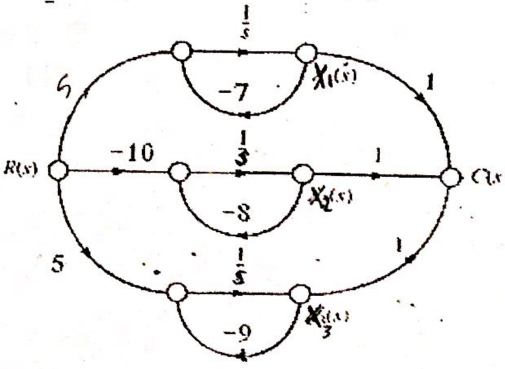
3- Parallel Form

$$\frac{C(s)}{R(s)} = G(s) = \frac{10}{s^3+24s^2+191s+504} = \frac{5}{s+7} - \frac{10}{s+8} + \frac{5}{s+9}$$

$= x_1 - x_2 + x_3$

$$\dot{x}_1 = -7x_1 + 5r$$
$$\dot{x}_2 = -8x_2 - 10r$$
$$\dot{x}_3 = -9x_3 + 5r$$

$$y = x_1 + x_2 + x_3$$



~~$G(s) = X_1 \cdot R - X_2 \cdot R + X_3 R$~~

$\frac{X_1}{R} =$

$$\dot{x} \begin{bmatrix} -7 & 0 & 0 \\ 0 & -8 & 0 \\ 0 & 0 & -9 \end{bmatrix} x + \begin{bmatrix} 5 \\ -10 \\ 5 \end{bmatrix} r \quad ,$$

$$y = \begin{bmatrix} 1 & 1 & 1 \end{bmatrix} x$$

Note

1- diagonal system matrix

2- Each equation is a first order diffrential equation is only one Variable.

3- The equations are said to be decoupled.

Controller Canonical form

- Another representation that uses Phase Variable is called the Controller Canonical form:-

- This form is obtained from the Phase Variable form by ordering the Phase Variable in the reverse order

$$G(s) = \frac{C(s)}{R(s)} = \frac{s^2 + 5s + 7}{s^3 + 3s^2 + 9s + 25}$$

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -25 & -9 & -3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} r$$

$$y = \begin{bmatrix} 7 & 5 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_1 \\ x_3 \end{bmatrix}$$

The Phase Variable in reverse order yields

$$\begin{bmatrix} \dot{x}_3 \\ \dot{x}_2 \\ \dot{x}_1 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -25 & -9 & -3 \end{bmatrix} \begin{bmatrix} x_3 \\ x_2 \\ x_1 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} r$$

$$y = \begin{bmatrix} 7 & 5 & 1 \end{bmatrix} \begin{bmatrix} x_3 \\ x_2 \\ x_1 \end{bmatrix}$$

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} -3 & -9 & -25 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} r$$

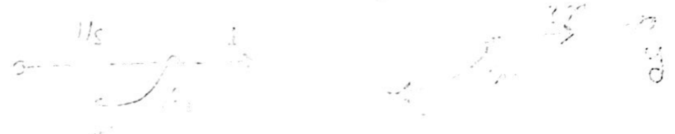
$$y = \begin{bmatrix} 1 & 5 & 7 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

$\frac{s+3}{[illegible]}$ [illegible] $\frac{1}{s+5}$ [illegible]

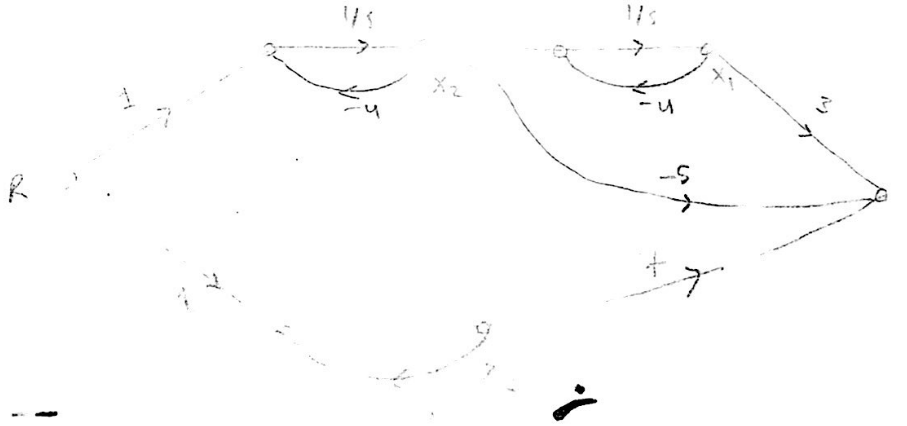
$\dot{x}_1 = -4x_1 + r$

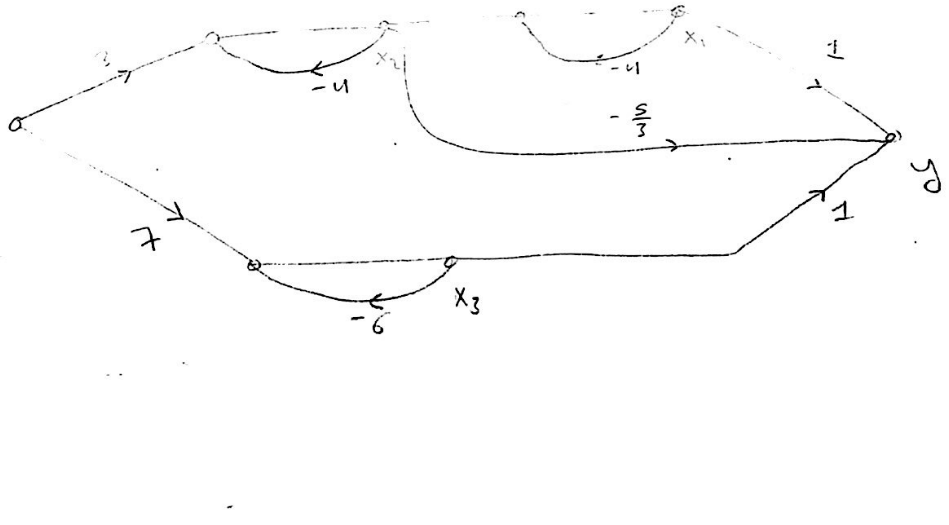
$\dot{x}_2 =$ [illegible]

y [illegible]



$\frac{Y}{R}$ [illegible] $\frac{3}{[illegible]}$ [illegible]





Home Work 4 ---------------- [illegible]

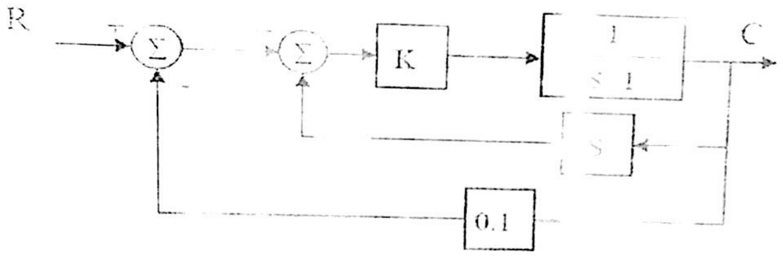
1- For the following systems :-

a- Reduce the block diagram in figures below to a single transfer function T(s)= C(s)/R(s).

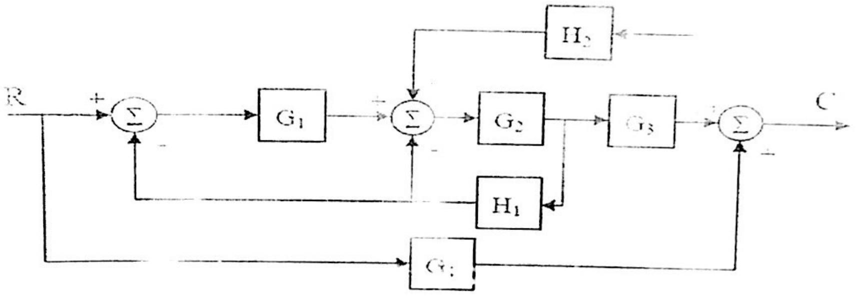
b- Draw a signal- flow graph for the block diagrams shown in the following figures.

c- Using Mason's rule, find the transfer function T(s)= C(s)/R(s) from SFG represented in part b.

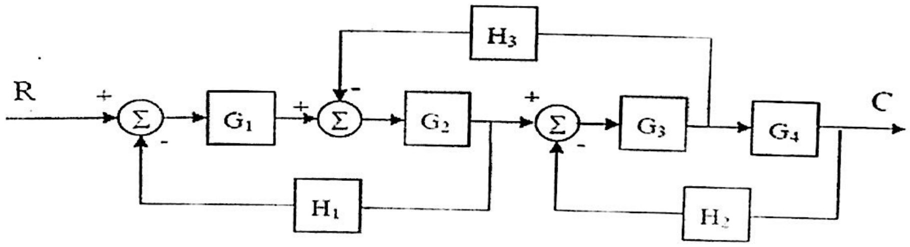
Case 1



Case 2



Case 3



2- Repeat case 2and 3 to find transfer function using MATLAB if

$$G1(s)=\frac{1}{s+1},\quad G2(s)=\frac{10}{s+5},\quad G3(s)=\frac{s}{s+5},\quad G4(s)=\frac{s+1}{s^2+s+1}$$

$$H1(s)=2,\quad H2(s)=\frac{1}{s},\quad H3(s)=s$$